3.2

Tickets for Sale

Standard Form of Linear Equations

LEARNING GOALS

In this lesson, you will:

- Identify contextual meaning of expressions in an function.
- Write equations in standard form.
- Solve equations in standard form.
- Determine the *x*-intercept and *y*-intercept of an equation in standard form.
- Use intercepts to graph an equation.
- Convert equations from standard form to slope-intercept form.
- Solve equations in slope-intercept form.
- Determine the *x*-intercept and *y*-intercept of an equation in slope-intercept form.
- Perform unit analysis of equations.

KEY TERMS

- standard form
- slope-intercept form

Have you ever wondered why many prices end in 0.99 instead of a multiple of 5 or 0? There are many theories on when and how this trend started. One theory is that in 1876 Melville E. Stone wanted to sell his newspaper for 1¢ to compete against other 5¢ newspapers. The problem was, not many people used pennies in those days. Stone convinced shopkeepers to set their prices to end in $0.99 so consumers would get the penny back as change. People then used their pennies to buy his newspapers and the pricing stuck. Another theory is that stores starting slightly lowering their prices to compete with other stores. Wouldn't you rather buy something that costs $2.99 at one store instead of $3.00 at another? One final theory is called psychological pricing. This states that consumers tend to ignore the least significant digits rather than do the proper rounding. We may see $19.^{99} as $19 instead of closer to $20, which is correct.

Do you agree with any of these theories? Can you think of any other reasons stores may set their prices to end in these amounts?

PROBLEM 1 How Much Did We Make?

The Marshall High School Athletic Association sells tickets for the weekly football games. Students pay $5 and adults pay $10 for a ticket.

1. How much money would the athletic association collect:
   a. if 100 students and 50 adults buy tickets to the game?

   b. if 125 students and 75 adults buy tickets to the game?

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   c. if 97 students and 116 adults buy tickets to the game?

2. Explain how you can determine the total amount of money collected if you know the number of student tickets sold and the number of adult tickets sold.

3. Write an expression to represent the total amount of money collected for any number of tickets sold. Use *s* to represent the number of student tickets sold, and *a* to represent the number of adult tickets sold.

4. Explain how your expression represents this problem situation.

The expression $5s + 10a$ can be written in function notation as a function of two variables.

$$f(s, a) = 5s + 10a$$

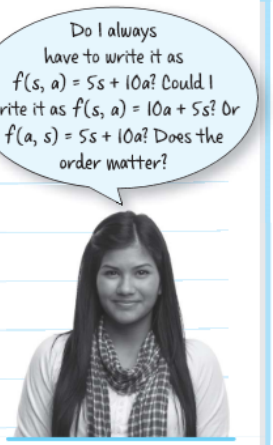
You can use this function to determine the amount of money collected if 75 student tickets and 45 adult tickets are sold.

$$f(75, 45) = 5(75) + 10(45)$$

$$f(75, 45) = 375 + 450$$

$$f(75, 45) = 825$$

The athletic association collected \$825 if they sold 75 student tickets and 45 adult tickets.



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5. What unit of measure describes $f(s, a)$? Explain your reasoning.

6. Use the function $f(s, a)$ to determine how much money is collected at each game.

a. Game 1
$f(85, 80)$

b. Game 2
$f(94, 63)$

c. Game 3
$f(54, 72)$

d. Game 4
$f(123, 108)$

7. If you know the number of student tickets sold, can you determine the total amount of money collected? Explain your reasoning.

8. If you know the total amount of money collected, can you determine the number of student and adult tickets sold? Explain your reasoning.

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PROBLEM 2 How Many Tickets Do We Need to Sell?

The football team is playing in an out-of-town tournament. The athletic association needs to raise $3000 to send the team to this tournament. The money raised from selling tickets to a special event home game will be used toward the tournament cost.

1. Write an equation to represent this problem situation using the same prices from Problem 1.

2. How does this equation differ from the function you wrote in Problem 1?

3. Use your equation to complete the table.

Quantity Name					
Unit					
Expression	s	a	$5s$	$10a$	$5s + 10a$
	250				
		150			
				3000	
	600				

4. Carla and Robena sell game tickets. They have already sold 95 student tickets. Carla says that they need to sell 252 adult tickets to reach the goal of $3000. Robena says that they need to sell 253 adult tickets to reach the goal. Who is correct? Explain your reasoning.

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PROBLEM 3 Analyzing Ticket Sales

Students that want to attend the special event game must purchase their tickets at school prior to the game. So far, 189 students bought tickets for the game. The athletic association wants to know how many adult tickets they must sell in order to reach their goal of \$3000. However, they want a method to make forecasting how many adult tickets they must sell more efficient.

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Another way to determine the number of adult tickets that must be sold to reach a goal of \$3000 is to transform the equation to isolate a first.

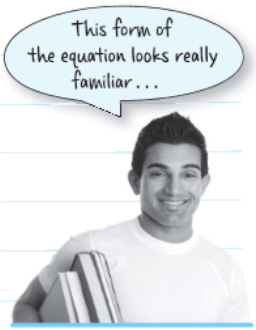
$$5s + 10a = 3000$$

$$5s - 5s + 10a = -5s + 3000$$

$$\frac{10a}{10} = \frac{-5s}{10} + \frac{3000}{10}$$

$$a = -\frac{1}{2}s + 300$$

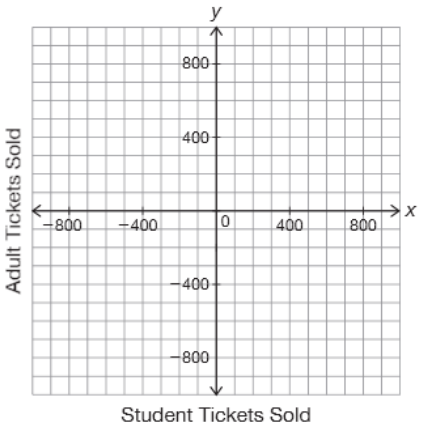
Now, substitute the information you know into the transformed equation.



1. How many adult tickets must be sold to reach the goal of \$3000?

2. Let the x-axis represent the number of student tickets sold. Let the y-axis represent the number of adult tickets sold.
   a. Determine the x-intercept and the y-intercept for the transformed equation. Explain what each intercept means in terms of the problem situation. What do you notice?

3. Use the *x*-intercept and *y*-intercept to graph the equation.



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4. Identify the slope of the graph. Interpret its meaning in terms of the problem situation.

5. The athletic association sold 400 student tickets. Determine how many adult tickets they must sell to reach the $3000 goal.

6. Can you use the graph to determine how much money is collected if the athletic association sold 400 student tickets and 200 adult tickets? Why or why not?

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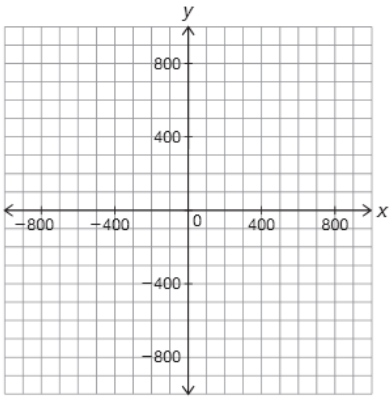
7. Let's consider reaching the $3000 goal for ticket sales by analyzing the number of adult tickets sold. If the association knows that 150 adult tickets have been sold, how many student tickets would they need to sell to reach their goal?

a. Transform the equation $5s + 10a = 3000$ to solve for the number of student tickets.

b. How many student tickets must the athletic association sell on homecoming weekend to reach their goal of $3000?

c. Determine the x-intercept and the y-intercept of the graph described by this equation. Explain what the intercepts mean in terms of the problem situation.

8. Use the x-intercept and the y-intercept to graph the equation.



9. Identify the slope of the graph. Interpret its meaning in terms of the problem situation.

10. Compare the *x*-intercepts and the *y*-intercepts of the two graphs you just created. What do you notice?

11. Is there a way to determine the total amount of money collected from either graph? Explain why or why not.

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You may have noticed that when you transformed the equations to isolate the *a* or *s* variables, the $3000 goal "disappeared." Where did the money go? Let's perform unit analysis on each part of one isolated *a* equation to see just where that $3000 went.

12. Identify the units of measure for each part of the equation, $5s + 10a = 3000$.

a. 5

b. *s*

c. 10

d. *a*

e. 3000

When you analyze the units of each part of the equation, you can see how the equation maintains balance.

$$5s + 10a = 3000$$

$$5 \cdot s + 10 \cdot a = 3000$$

$$\frac{\text{dollars}}{\text{student tickets}} \cdot \text{student tickets} + \frac{\text{dollars}}{\text{adult tickets}} \cdot \text{adult tickets} = \text{dollars}$$

$$\frac{\text{dollars}}{\cancel{\text{student tickets}}} \cdot \cancel{\text{student tickets}} + \frac{\text{dollars}}{\cancel{\text{adult tickets}}} \cdot \cancel{\text{adult tickets}} = \text{dollars}$$

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13. Write the next sentence in the worked example after dividing out the two different units of measure. What does this tell you about the original equation?

When you rewrote the equation to determine the number of adult tickets sold, the units were rearranged.

When you transformed the equation to determine the number of adult tickets sold, the units were also rearranged.

$$5s + 10a = 3000$$

$$10a = -5s + 3000$$

$$\frac{10a}{10} = \frac{-5s}{10} + \frac{3000}{10}$$

$$a = -\frac{1}{2}s + 300$$

Let's consider the units of each part of the highlighted equation to show how they simplify:

$\frac{10a}{10}$ simplifies to a

10 a

$\frac{\text{dollars}}{\text{adult tickets}} \cdot \frac{\text{adult tickets}}{\text{dollars}}$ $\frac{\text{adult tickets}}{10}$

$\frac{\text{dollars}}{\text{dollars}}$ − dollars · $\frac{\text{adult tickets}}{\text{dollars}}$ adult tickets

$\frac{-5s}{10}$ simplifies to $-\frac{1}{2}s$

−5 s

$\frac{\text{dollars}}{\text{student tickets}} \cdot \frac{\text{student tickets}}{\text{adult tickets}}$ $\frac{\text{adult tickets}}{10}$

$\frac{\text{dollars}}{\text{dollars}}$ − dollars · $\frac{\text{adult tickets}}{\text{dollars}}$ adult tickets

$\frac{3000}{10}$ simplifies to 300

3000

$\frac{\text{dollars}}{\text{adult tickets}}$ $\frac{\text{adult tickets}}{10}$

dollars · $\frac{\text{adult tickets}}{\text{dollars}}$ adult tickets

So, in $a = -\frac{1}{2}s + 300$, all the units are balanced.

adult tickets = adult tickets + adult tickets

Linear functions can be written in two forms, *standard form* and *slope-intercept* form.

The **standard form** of a linear equation is $Ax + By = C$ where A, B, and C are constants and A and B are not both zero.

In the first worked example, the original equation was written in standard form with the dollars units already isolated on one side of the equation.

The **slope-intercept form** of a linear equation is $y = mx + b$ where b is the y-intercept and m is the slope.

To isolate the a-variable in the second worked example, you transformed the original equation written in standard form to the slope-intercept form with the a-variable representing the dependent quantity.

As you have seen in this problem situation, it is often necessary to change between the forms as one form might be more appropriate to represent a situation.

14. Explain what happened to the units of students and dollars when converting from standard form to slope-intercept form in the worked example.

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15. Convert the standard form of the original equation $5s + 10a = 3000$ to slope intercept form to represent the number of student tickets. Show and explain the final units for the equation.

Talk the Talk

Katie received a $75 gift card for her birthday. She decides to buy new music and movies for her electronic notebook with the gift card. Songs cost $1.29 each and movies cost $14.99 each.

1. Write an equation to represent this problem situation. Use *s* to represent the number of songs and *m* to represent the number of movies.

2. Complete the table to show what each expression represents in this problem situation.

Expression	Contextual Meaning
s	
m	
1.29	
14.99	
$1.29s$	
$14.99m$	
$1.29s + 14.99m$	
75	

3. If Katie buys 20 songs, what is the greatest number of movies she can buy?

4. If Katie buys no movies, what is the greatest number of songs she can buy? What does this number represent?

5. If Katie buys no songs, what is the greatest number of movies she can buy? What does this number represent?

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Be prepared to share your solutions and methods.